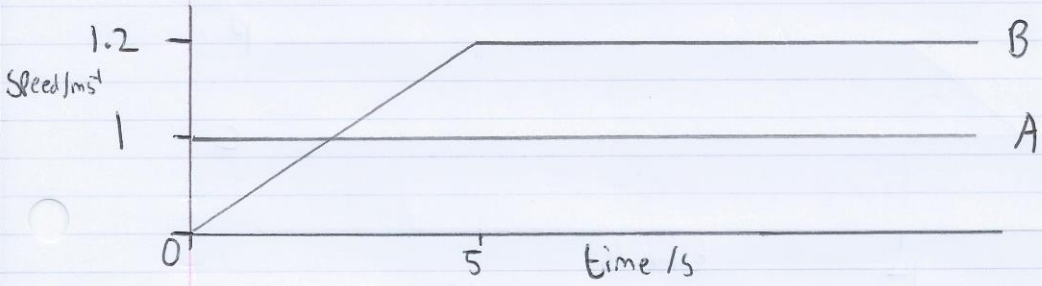
Overtaking exercise 1

1 Object A leaves a point O with speed $1\,ms^{-1}$ and maintains a constant speed.
At the same time, Object B leaves point O but has an initial velocity of $0\,ms^{-1}$ and accelerates up to a constant speed of $1.2\,ms^{-1}$ after 5 seconds.

Find the time at which object B overtakes A.



Both speed & time will be equal at overtake point

* Acceleration of B: $\frac{v-u}{t} = \frac{1.2}{5} = 0.24\,ms^{-2}$

– Checking to see if overtake occurs before $t = 5$ seconds

$S_A = 1 \times 5 = 5\,m$ $\quad S_B = \frac{1}{2}(1.2+0) \times 5 = 3\,m$ (so still behind A)

$\therefore$ Overtake occurs when $t > 5$ s

$\therefore$ Let $t = 5 \longrightarrow t = 0$

Object A | $u=1$ $v=1$ $a=0$ $t=?$ $s=?$ | $u=1.2$ $v=1.2$ $a=0$ $t=?$ $s=?$

initial distance travelled: $S + (1 \times t) = 3 + 1.2 \times t$ (initial distance travelled)

$\therefore \quad 5 + t = 3 + 1.2t$

$2 = 0.2t$

$10 = t$

So Σ time $= 10 + 5$ ("time previously")

$t = 15$ seconds

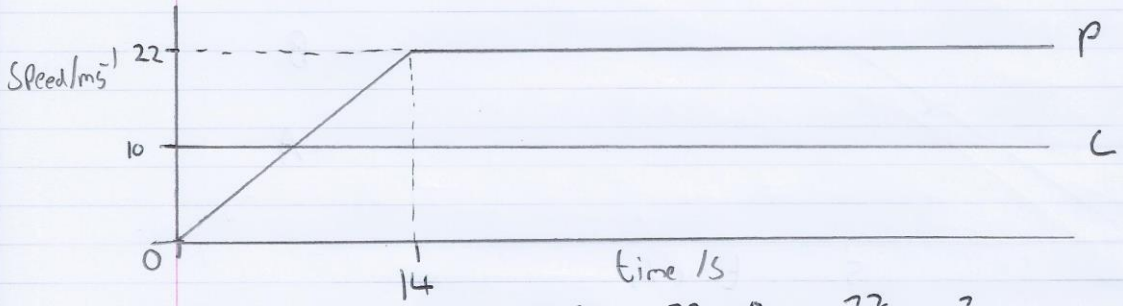
NOTE: Alternatively you can use $\frac{\Delta D}{\Delta V} = \Delta T$ so $\frac{(5-3)}{1.2-1} = 10$ seconds

Σ time $= 10 + 5 = 15$ seconds ✓

Overtaking example 2

2 A car P is stationary at a set of traffic lights. The lights change, $t = 0$, and the car accelerates from rest to $22\ ms^{-1}$ in 14 s.
A cyclist, C, travels at a constant speed of $10\ ms^{-1}$ through the traffic lights at $t = 0$ s (the instant the car begins to move).

Find the time when the car, P, overtakes the cyclist, C.



* Acceleration of P $= \frac{v-u}{t} = \frac{22-0}{14} = \frac{22}{14}\ ms^{-2}$
* Check to see if the overtake occurs when $t < 14$ s, (which it does since the area shown for P is $>$ that of C. However, this step is still necessary

$S_P = \frac{1}{2}(22+0) \times 14$ $\qquad$ $S_C = 10 \times 14$
$= 154$ m $\qquad$ $= 140$ m
$S_P > S_C$ so overtake occurs before $t = 14$ s

* Distance, S, & time, t, will be identical for both bodies

∴ Car, P : Cyclist, C

$S_P = ut + \frac{1}{2}at^2$ $\qquad$ $S_C = ut$

$0 + \frac{1}{2}\left(\frac{22}{14}\right)t^2 = 10t$

$\frac{11}{14}t^2 = 10t$

$11t^2 - 140t = 0$

$t(11t - 140) = 0$

$t = 0$ or $t = \frac{140}{11}$

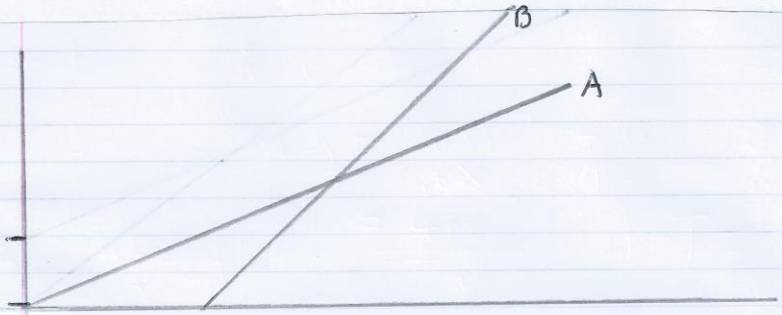
* this is the time when they are next to each other at the traffic lights initially

$t = 12.72$ seconds
$t = 12.7$ seconds (3 s.f.)

Overtaking example question 3

6. Two cars A and B are moving in the same direction along a straight horizontal road. Car A is moving with uniform acceleration $0.4\,\text{m s}^{-2}$ and car B is moving with uniform acceleration $0.5\,\text{m s}^{-2}$. At the instant when B is 200 m behind A, the speed of A is $35\,\text{m s}^{-1}$ and the speed of B is $44\,\text{m s}^{-1}$. Find the speed of B when it overtakes A.

(9)



CAR B | $a = 0.5\,\text{m s}^{-2}$ $u = 44\,\text{m s}^{-1}$ & is 200m behind A

CAR A | $a = 0.4\,\text{m s}^{-2}$ $u = 35\,\text{m s}^{-1}$

At the point where B overtakes A, time is identical and so is the net distance travelled by each.

∴ CAR A : CAR B

$s = ut + \frac{1}{2}at^2$ (+200) : $s = ut + \frac{1}{2}at^2$ (+0)

Since A is 200m ahead of B

$$\therefore \quad 200 + 35t + \tfrac{1}{2}(0.4)t^2 = 44t + \tfrac{1}{2}(0.5)t^2$$

$$0.20t^2 = 0.25t^2 + 9t - 200$$

$$0.05t^2 + 9t - 200 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$t = 20$ ✓ or $t = -200$ ✗ (can't get negative time)

So time = 20 seconds

So speed of B, $= u + at$

$= 44 + 0.5(20)$ $= 54\,\text{m s}^{-1}$ ✓

Overtaking example 4

5. A car and a motorbike are at rest adjacent to one another at a set of traffic lights on a long, straight stretch of road. They set off simultaneously at time $t = 0$. The motorcyclist accelerates uniformly at 6 ms^{-2} until he reaches a speed of 30 ms^{-1} which he then maintains. The car driver accelerates uniformly for 9 seconds until she reaches 36 ms^{-1} and then remains at this speed.

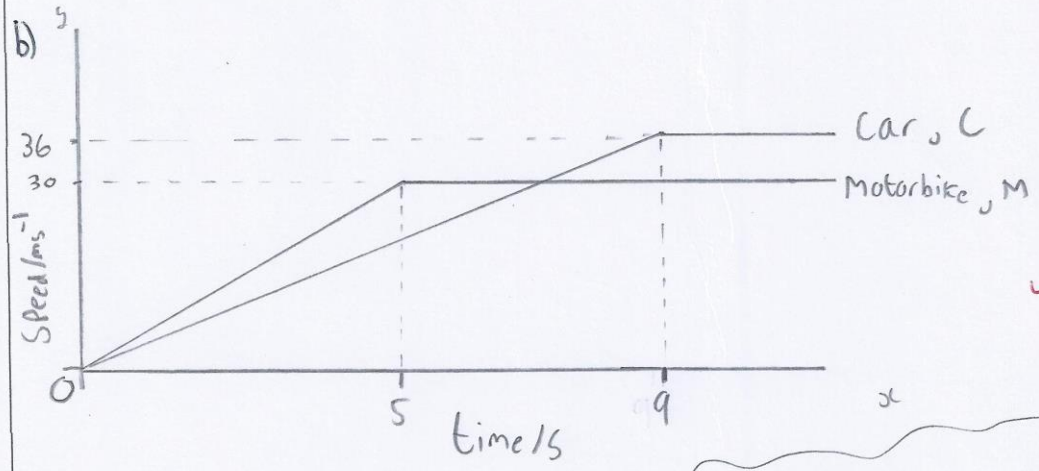
(a) Find the acceleration of the car. **(2 marks)**

(b) Draw on the same diagram speed-time graphs to illustrate the movements of both vehicles. **(4 marks)**

(c) Find the value of t when the car again draws level with the motorcyclist. **(7 marks)**

a) $\frac{v-u}{t} = a \quad \frac{36-0}{9} = 4ms^{-2}$ ✓

b)



✓

c) Does the overtake occur < 9 seconds?

$S = \frac{1}{2}(v+u)t$ & $S = vt$ (constant)

Distance of Car, C	Distance of motorbike, M
$S_c = \frac{1}{2}(36+0)9 = 162m$	$S_m = \frac{1}{2}(30+0)5 + (30 \times 4)$
	$S_M = 75 + 120$
	$S_M = 195m$

∴ Car has not yet overtaken the bike

So let $t = 9 \longrightarrow t = 0$

Speed of C = 36 ms^{-1} Speed of M = 30 ms^{-1}

Distances will be identical & times identical on overtake

∴ $\underbrace{162 + 36(t)}_{S \text{ for Car}} = \underbrace{195 + 30(t)}_{S \text{ for M}}$

$36(t) = 30(t) + 33$

$6t = 33$

$t = 5.5$ seconds (after $t = 9$ seconds)

∴ $T = 5.5 + 9 = 14.5$ seconds

∴ T = 14.5 seconds

✓